Logic: Karnaugh Maps

The Karnaugh Map provides a simple means of simplifying certain classes of Boolean expressions. In this document we consider the situation of having four Boolean expressions like the following one:

$$\overline{A}.\overline{B}.C.D+\overline{A}.B.C.\overline{D}+\overline{A}.B.C.D+A.\overline{B}.C.D+A.B.C.\overline{D}+A.B.C.D$$

For the Karnaugh map we interpret them in order, but each letter on its own is taken to be a “1” and each letter with a bar above is taken to be a “0”. So the expression above becomes:

0011,0110,0111,1011,1110,1111

First we draw a grid as shown below with the combinations of AB values running right and the combinations of CD running down the grid. Notice that it is the practice to put the combinations in Gray code, that is in the order 00, 01, 11, 10; that is only one bit changes at a time.

CD\AB	00	01	11	10
00				
01				
11				
10				

In a Karnaugh map, we place a “1” for each component. For example $\overline{A}.\overline{B}.C.D$ or “0011” results in is placing a “1” in the “00” column and “11” row:

CD\AB	00	01	10	11
00				
01				
11	1			
10				

Continuing with the other components gives the following completed Karnaugh map.

CD\AB	00	01	11	10
00				
01				
11	1	1	1	1
10		1	1	

In order to simplify the original expression, we first try to draw lozenge-shaped figures around the groups of 1s, as shown in the following diagram. It does not matter if the lozenges overlap.

CD\AB	00	01	11	10
00				
01				
11	1	1	1	1
10		1	1	

We now consider each lozenge in turn. For each lozenge we consider which variables do no change and this is the Boolean expression that simplifies the expression that is represented by that group of 1s.

For example for the long thin lozenge, below

CD\AB	00	01	11	10
00				
01				
11	1	1	1	1
10		1	1	

we note that C and D do not change and are both “1” so this simplifies to C.D.

For the square lozenge

CD\AB	00	01	11	10
00				
01				
11	1	1	1	1
10		1	1	

we note that B=”1” and C=”1” and so this simplifies to B.C.

Hence $\overline{A}.\overline{B}.C.D+\overline{A}.B.C.\overline{D}+\overline{A}.B.C.D+A.\overline{B}.C.D+A.B.C.\overline{D}+A.B.C.D=C.D+B.C$

For a second example we will simplify

$$\overline{A}.\overline{B}.C.D+\overline{A}.B.C.\overline{D}+\overline{A}.B.C.D+\overline{A}.\overline{B}.\overline{C}.D+\overline{A}.\overline{B}.C.\overline{D}+\overline{A}.\overline{B}.\overline{C}.\overline{D}$$

Using binary notation this is equivalent to

0011, 0110, 0111, 0001, 0010, 0000.

The Karnaugh map is as follows:

CD\AB	00	01	11	10
00	1			
01	1			
11	1	1		
10	1	1		

For the vertical column A and B are always both “0”, so this simplifies to $\bar{A}.\bar{B}$. For the square lozenge we note that A is always “0” and C is always “1”, so this simplifies to $\bar{A}.C$.

Hence $\overline{A}.\overline{B}.C.D+\overline{A}.B.C.\overline{D}+\overline{A}.B.C.D+\overline{A}.\overline{B}.\overline{C}.D+\overline{A}.\overline{B}.C.\overline{D}+\overline{A}.\overline{B}.\overline{C}.\overline{D}=\overline{A}.\overline{B}+\overline{A}.C$