Introduction to Logic

In this document we consider the concepts and rules of Logic or Boolean Algebra. All computer systems are based on logic. Computers are ultimately based on 1s and 0s or *true* (T) or *false (F)* values*.* Logical statements are based on *propositions and* the resulting *propositional logic* is introduced in this document

Propositional Logic

A proposition is a statement that is either true (T) or false (F).

For example: *A cat is an animal* is true. A proposition that is true is a *fact.*

For example: *3 = 4* is false.

Propositions can be linked using a *connective* such as "*and*" or "*or*" to form a compound proposition*.* The definitions of "*and*" or "*or*" in logic conform to our everyday use.

For example: *A cat is an animal and a flower is a plant.*

For example: *3<7 or 3>4 .*

If a compound proposition is joined by the "*and*" connective then it is true if both propositions are true, and false otherwise. If a compound proposition is joined by the "*or*" connective then it is true if either proposition is true, and false only if both are false.

For example: *A cat is an animal and a flower is a plant* is *true* since both propositions *"a cat is an animal"* and "*a flower is a plant"* are *true*.

For example: *3<7 or 3>4* is *true* even though only one of the propositions is *true*.

The connectives "*and"* and "*or"* are *logical operators*. They are also binary operators since they connect two logical statements. The other most important logical operator *"not"* – it is a unary operator since it acts on only one logical statement.

A *logical statement* is made up of one or more linked propositions and it evaluates to *true* or *false*. For convenience we may use a letter to represent a logical statement.

For example by letting A= "3<7" and B= "3>4" then the logical statement *3<7 or 3>4* can be written simply as A *or* B.

A logical statement can imply another logical statement. For example $x > 5$ implies that $x > 3$.

A logical statement can be equivalent to another logical statement. For example $x \geq 5$ and $x \leq 5$ is equivalent to (that is it implies and is implied by) the statement $x = 5$.

The *"and"* operation is also called *logical conjunction*, the *"or"* operation is also called *logical disjunction* and the *"not"* operation is also called *logical negation*.

Notation

Unfortunately, there seems to be a number of notations used in logical expressions. The most common are listed below.

not A	¬A	$\overline{A}$
A *and* B	A∧B	A.B
A *or* B	A∨B	A+B

For implication and equivalence, we have the following notation.

A *implies* B	A⇒B
A *is equivalent to* B	A⇔B

We can also use a functional notation to aid in creating statements.

For example :

Animal(cat) meaning that a cat is an animal

Plant(flower) meaning that a flower is a plant

Greater(7,3) meaning that 7 is greater than 3

However, the statements are specific; the extra structure of *predicate logic* is required in order to make general statements.